On the Monotonicity and Linearity of Ideal Radix-based A/D Converters

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A/D Converters

- Nyquist-rate converters
  - flash
  - sub-ranging
    - N-bit/stage, 1-bit/stage
    - two-stage, pipeline, cyclic
  - successive approximation
  - integrating (E.g. dual-slope)
- Oversampling converters
  - simple oversampling
  - noise shaping ($\Delta\Sigma$ or $\Sigma\Delta$ converters)

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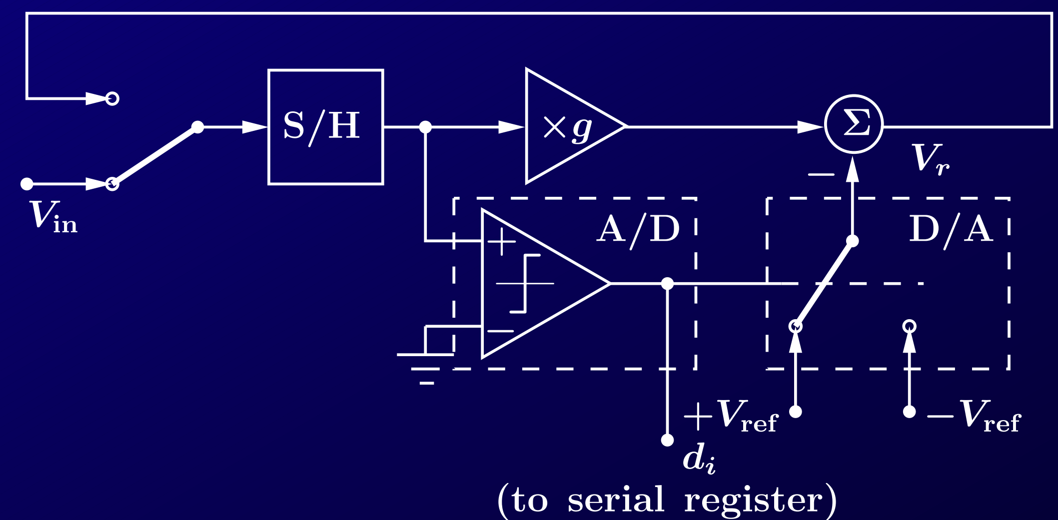
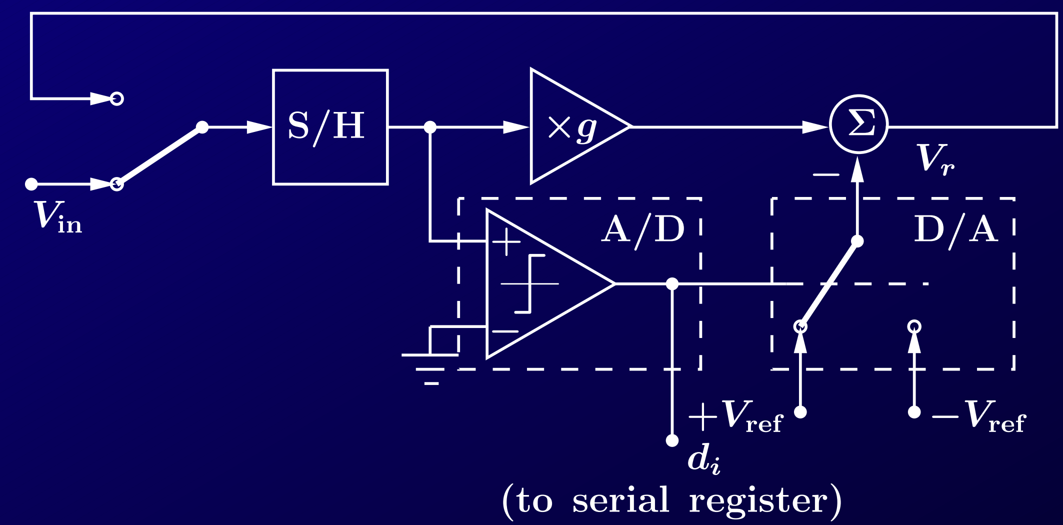
Cyclic A/D Converters



$g = 2$ **in the ideal case.**

$$
\begin{aligned}
V_r[1] &= gV_{in} - d_1 V_{ref} \\
V_r[2] &= gV_r[1] - d_2 V_{ref} = \\
&= g^2 V_{in} - g d_1 V_{ref} - d_2 V_{ref}
\end{aligned}
$$

Cyclic A/D Converters



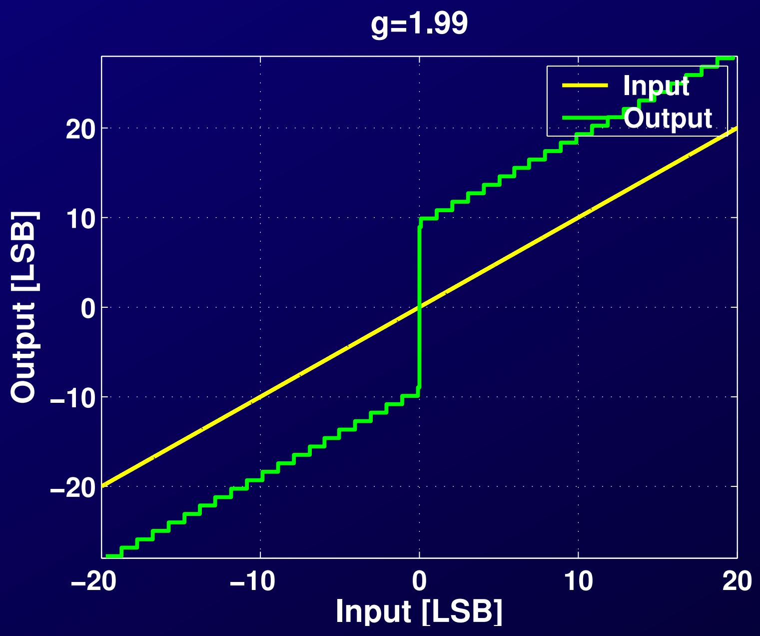
$$\vdots$$

$$\begin{aligned} V_r[n] &= g^n V_{in} - g^{n-1} d_1 V_{ref} - \ldots - d_n V_{ref} = \\ &= g^n \left(V_{in} - \sum_{i=1}^{n} g^{-i} d_i V_{ref} \right). \end{aligned}$$

Errors in Cyclic A/D Converters

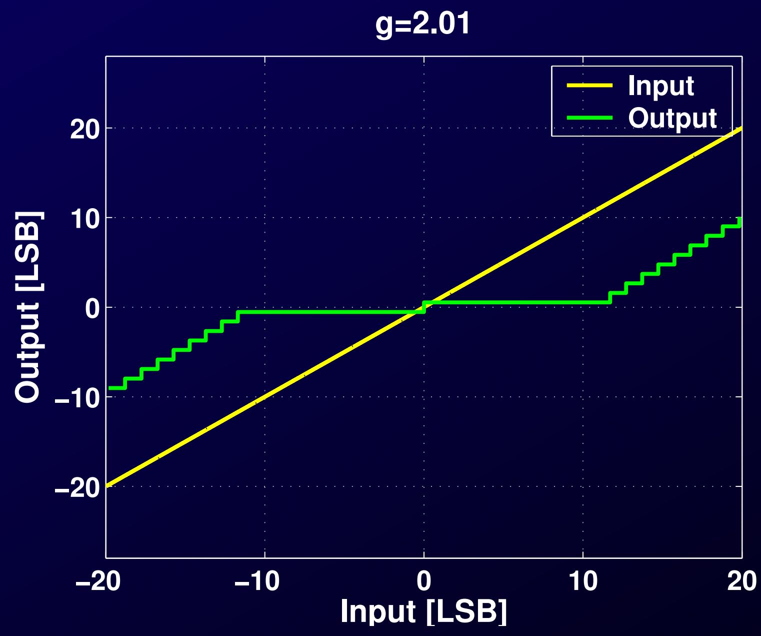
$g < 2$

Missing codes



$g > 2$

Missing decision levels



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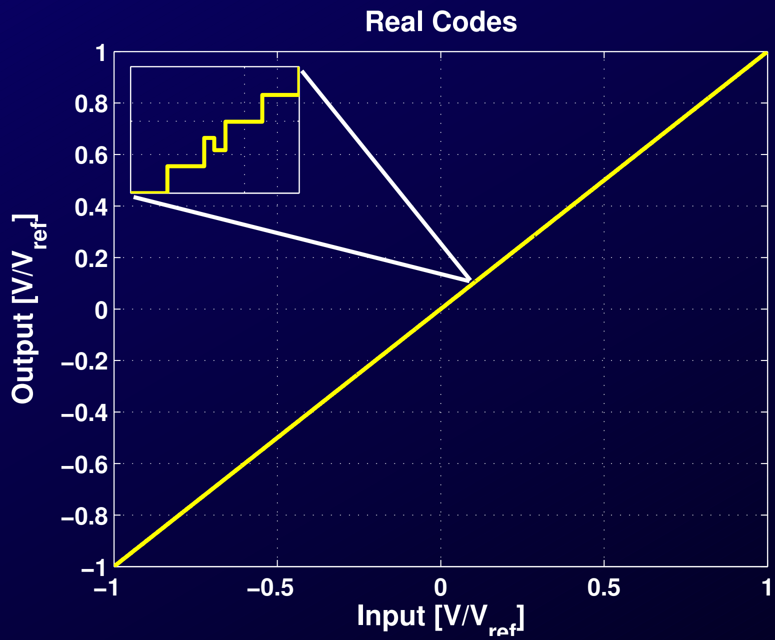
Digital Calibration

- Use $\hat{g}$ in the digital output [6]:
  $$\sum_{i=1}^{n} \hat{g}^{-i} d_i$$
- Not one-to-one mapping ⇔ Missing codes.
- Example:
  $$g = \hat{g} = 1.95$$
- Non-monotonic jumps in the output!



Non-monotonic Jumps

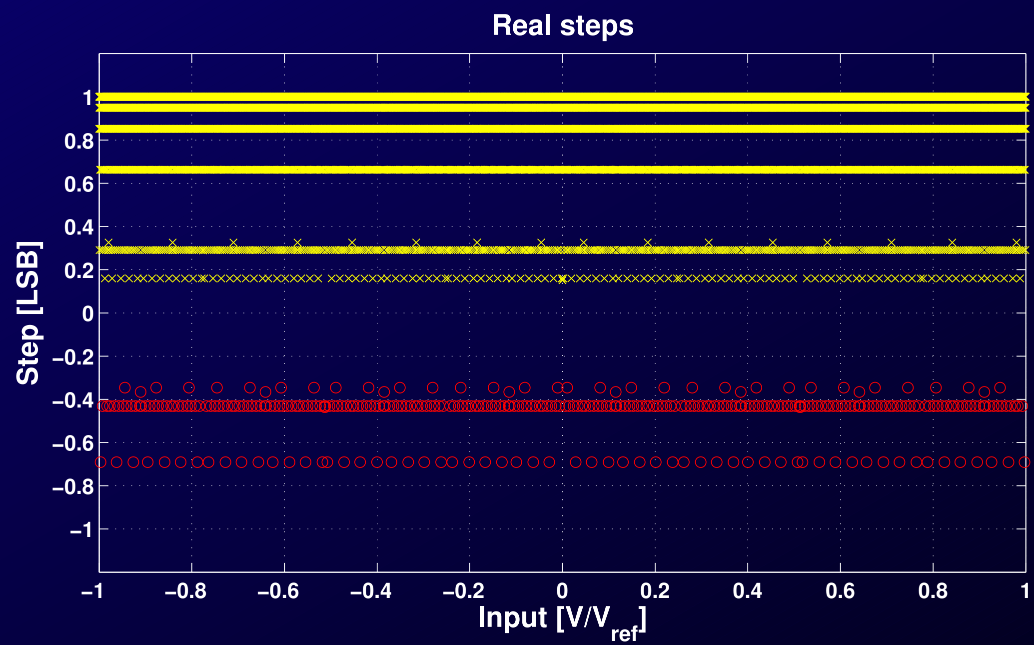
- **Due to the operation (cf. slide 2):**

$$\left| V_{\text{in}} - \sum_{i=1}^{n} g^{-i} d_i V_{\text{ref}} \right| = \frac{|V_r[n]|}{g^n} \le \frac{V_{\text{ref}}}{g^n}.$$

- **The same holds for adjacent codes**
- **E.g. MSB-changes** $(0\,\underbrace{111\ldots1}_{k} \Rightarrow 1\,\underbrace{000\ldots0}_{k})$
  - **To be solved for** k: $-1 < g^k - \sum_{i=0}^{k-1} g^i < 0$
  - **E.g. if** $g = 1.95$ **then** $k = 5$ $(-0.431$ **LSB)**
  - **There always exists such a** k **for** $g \in (1, 2)$

Number of Errors I.

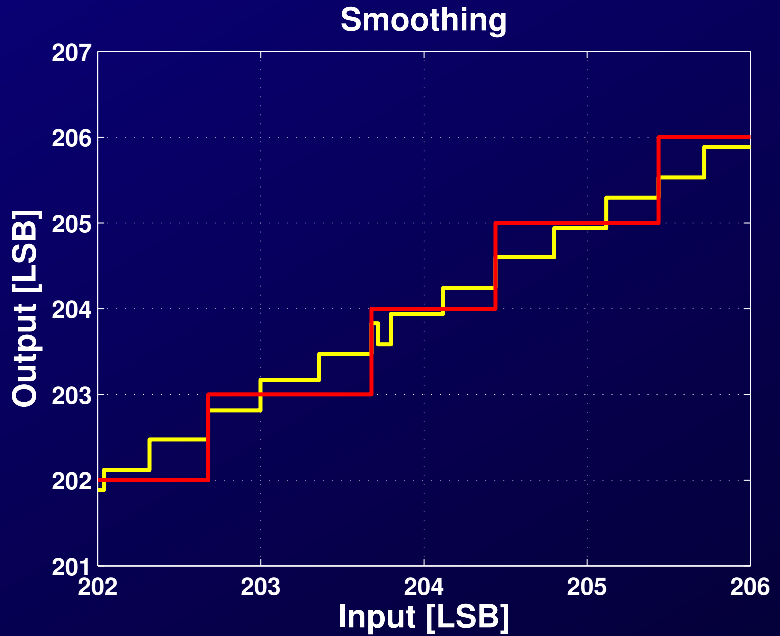
- If $g = 1.95$ and $n = 14$, then the number of negative steps are: 326 $(1.49\%$ of input range).



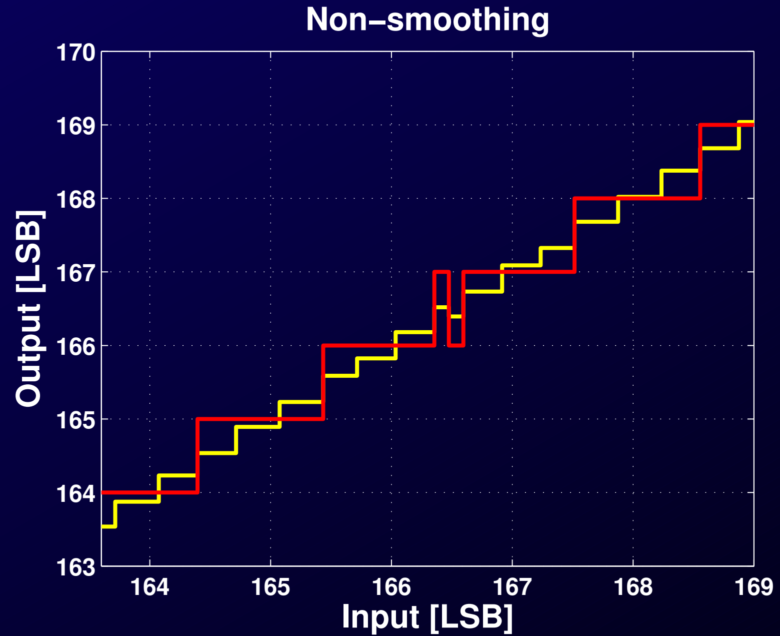
- As $g^n < 2^n$ $(11494 < 16384)$, requantization $(n_{\text{bit}} \leq n - 2)$ is required.

Requantization

Smoothed negative step



Non-smoothed negative step



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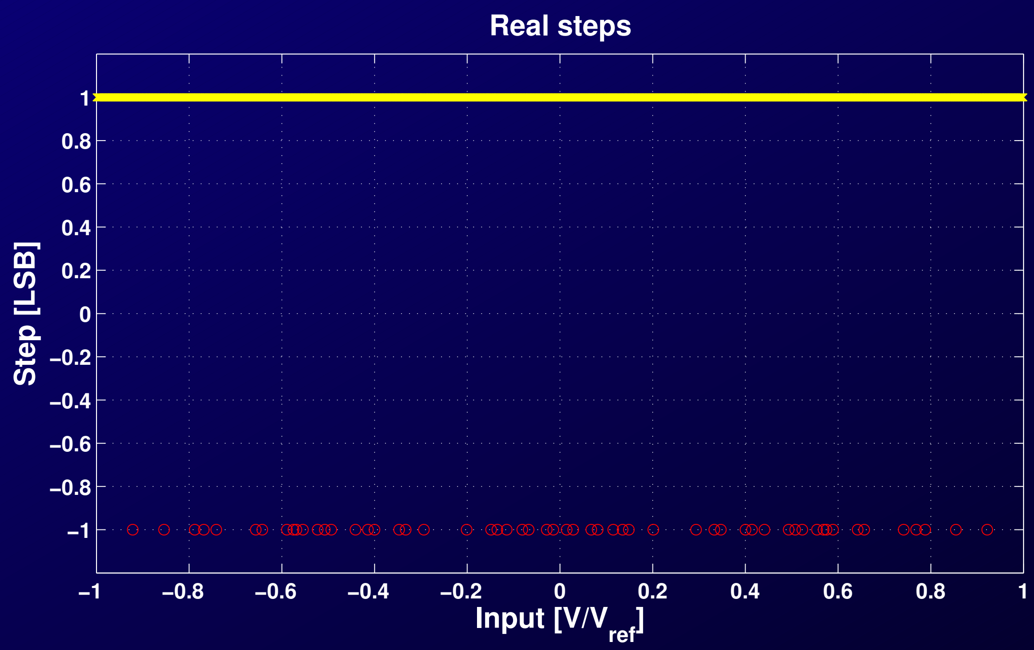
Number of Errors II.



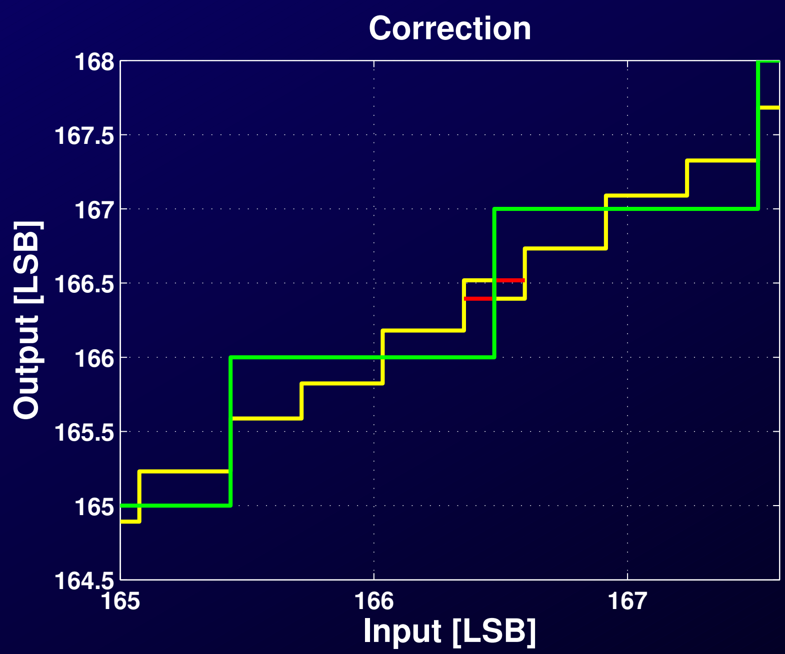
- $g = 1.95$, $n = 14$, $n_{\mathrm{bit}} = 12$
- **Remaining non-monotonic steps: 56 (0.84% of input range)**

Ensuring Monotonicity

- g-depending k
- $\ldots 0\underbrace{111\ldots 1}_{k}$ code:

  add $+1$
- $\ldots 1\underbrace{000\ldots 0}_{k}$ code:

  add -1
- The two non-monotonic codes are exchanged.



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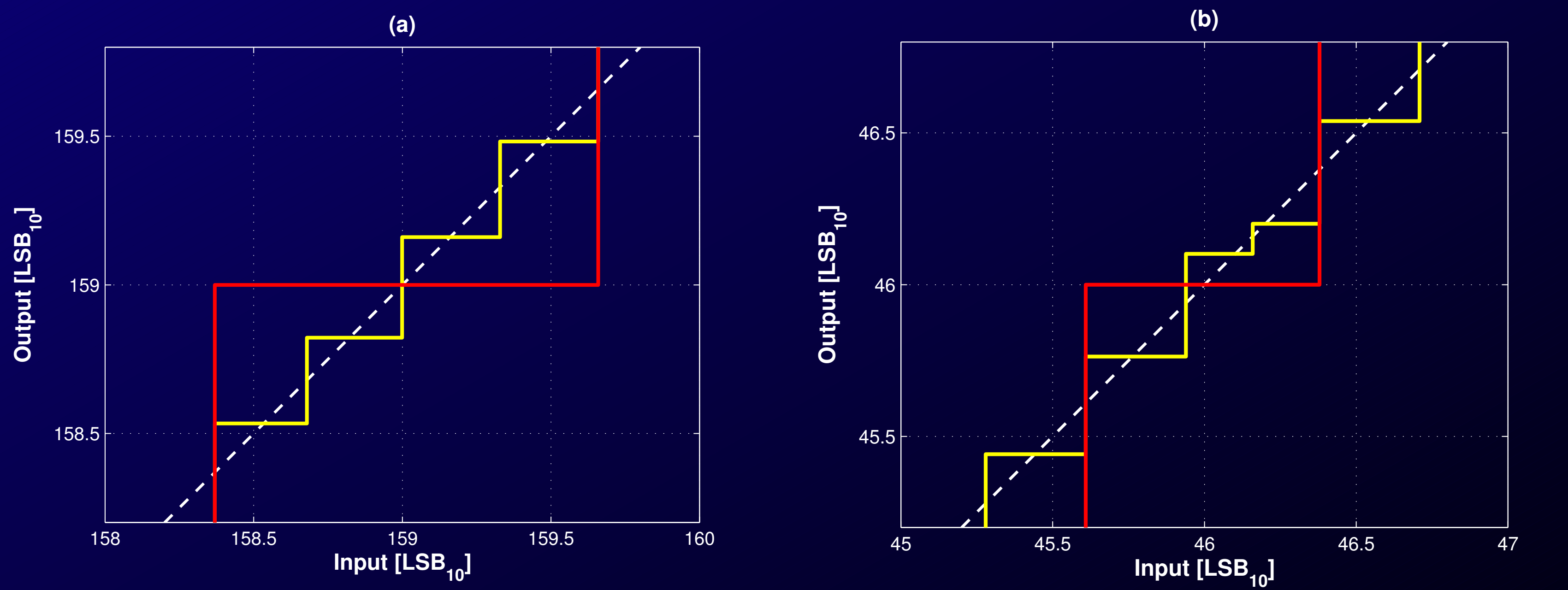
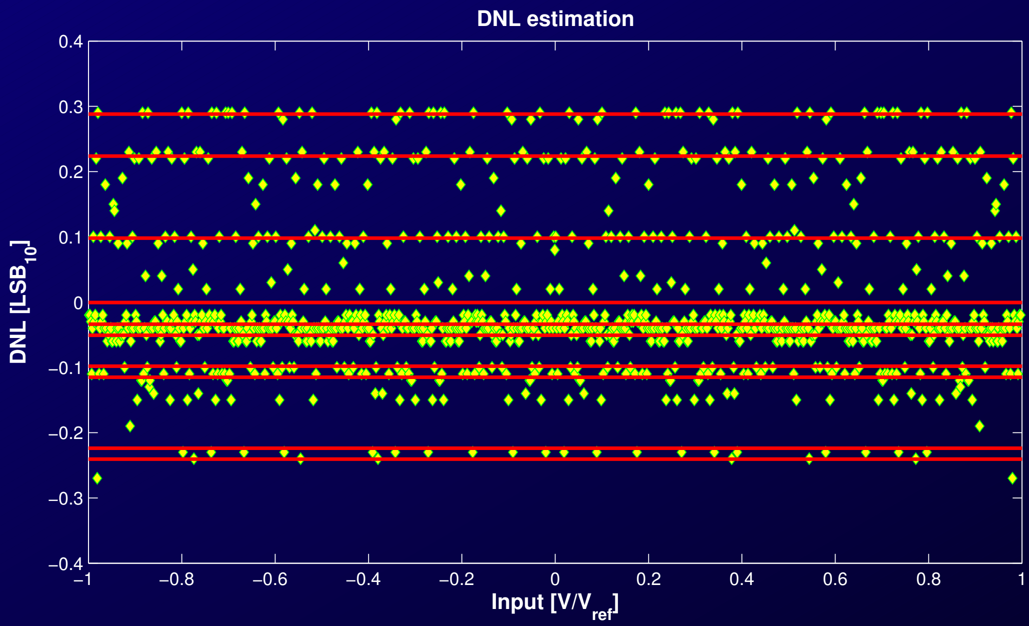
Linearity of the Converters

Positive differential non-linearity (DNL > 0)

Negative differential non-linearity (DNL < 0)



Estimation of the DNL



- $g = 1.95$, $n = 12$, $n_{\mathrm{bit}} = 10$
- **DNL is represented by markers**

Conclusions

- Cyclic A/D converter
  - operation
  - typical errors
- Digital calibration
- Non-monotonic transitions
- Ensuring monotonicity
- Linearity of the converter

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Thank you for your attention!